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DESIGN OF PARALLEL DIGITAL SYSTEMS AND SEMANTIC TRAPS

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Abstract

Digital systems will be specified with the help of finite state machines (FSM). Parallel digital systems can be specified with the help of finite state machines as well. Their specification leads to a system of FSM's. The system function is just implicitly noted now. It means in the design semantic traps are hidden. Especially Mealy-machine systems are discussed more in detail. A special view is dedicated to the problem of deadlock in parallel digital systems. Design recommendations can be derived in order to avoid unintentional asynchronous behavior. Moreover a mathematical basis for deadlock analysis of parallel digital systems is suggested.

1 INTRODUCTION

Digital systems are designed on the basis of a functional notation by a structural synthesis or by direct structural notation. Even with relatively small systems and low complexity it shows that systems can be described heavily by only one automaton, only a single FSM. In the result of the design a system of FSM's develops now. The function of the separate components is explicitly noted in the well known way. The difficulty thereby consists of overlooking the total behaviour of the system of FSM's, what leads into a dilemma for the designer. On the one hand he aims at a system with a semantically correct function. However on the other hand he is only in the position to note this function implicitly. I.e., the structure and the function of the individual modules and their coupling among themselves as well as with the environment are noted. In the following problems in systems of FSM's under syntactic, semantic and pragmatic aspects are to be discussed. The transition to an explicit system function is possible in principle by composition, but leads to a pragmatic barrier however. Complex functions extend over a lot of states as well as input and output variables. Out of that an unacceptable order of magnitude of functional models in the case of digital problems results. A compostion is practically not feasible for the aforementioned quantitative reasons. Nevertheless this path is taken here to make a statement about the behavior of structural noted systems, about the caused sematic traps or about possibilities for a meaningful a priori structuring at least in principle.

2 BASICS

The starting point is the concept of the finite state machines. Generally a finite state machine is defined as:

Definition 1: *A finite state machine is a tuple*

$$A = [X, Y, Z, \delta, \lambda] \tag{1}$$

with X - set of inputs, Y - set of outputs, Z - set of states, the state transition function,

$$\delta : X \times Z \to Z \tag{2}$$

the Mealy-output function

$$\lambda : X \times Z \to Y \tag{3}$$

and the Moore-output function

$$\mu : Z \to Y \tag{4}$$

Figure 1 shows the general structure of a clocked synchronous state machine, with c as a clock signal. It is shown the Mealy-machine. In practice the τ block is a set of n flip-flops that stores the current state of the machine, and has 2^n distinct states. The flip-flops are all connected to the common clock signal c that causes the flip-flops to change their state at each tick of the clock. The output function λ determines the output as a function of the current state and input. Remember, both δ and λ blocks are strictly combinational logic functions.

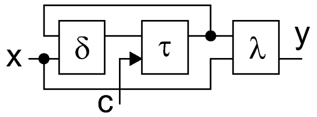

Figure 1: Clocked synchronous state-machine structure (Mealy-machine)

As a result of the composition of the FSM system components A^i with $A^i = [X^i, Y^i, Z^i, \delta^i, \lambda^i]$ a global automaton is derived.

Definition 2: *A global automat is a tuple*

$$A = [X, Y, G, \delta, \lambda] \tag{5}$$

G - set of global states, Z^i - state of the i-th finite state machine n - number of finite state machines

$$G = \prod_{i=0}^{n-1} Z^i. \tag{6}$$

3 DESIGN

A general coupling between two Mealy-machines can be done by states and/or by the outputs. Automaton systems, which are coupled only during states, that means only during the δ blocks, are regular structures. The number of states out of the composition is the product of the number of states for all system components, instead of semantic redundant states.

A measuring setup is given in accordance to figure 2, for the determination of the twist of a rotary flexible axis. Arbitrary directions of rotation are also possible as well as oscillations of the system. The measuring procedure is based on the fact that incremental sensors are fastened to the ends of the flexible part of the axis, allowing the evaluation of the impulses. The algorithm to be designed has to control a forward (v) - backward (r) - counter correctly, so that the twist is readable on-line at the counter. The simultaneous occurence of v and r impulses has to be prevented.

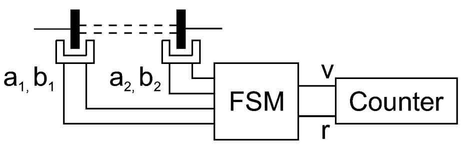

Figure 2: Example: measuring setup

It is possible to design a finite state machine with the inputs a_1, b_1, a_2, b_2 and the outputs v, r easily. The behavior is specified by the following state diagram in figure 3.

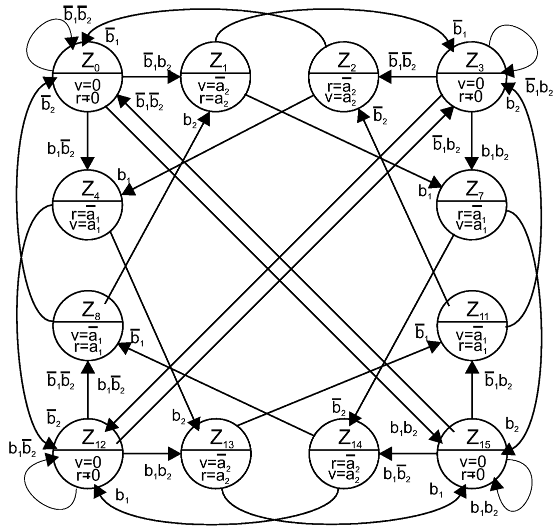

Figure 3: Example: gobal automat

It can be observed, that in spite of the consciously chosen low complexity of the example the specification by the automaton graph can be quite complex though. Practically relevant problems can be specified usually only as systems of FSM's. Figure 4 shows a specification of such a system of FSM's for the above example with the same system behavior as the state diagram from figure 3.

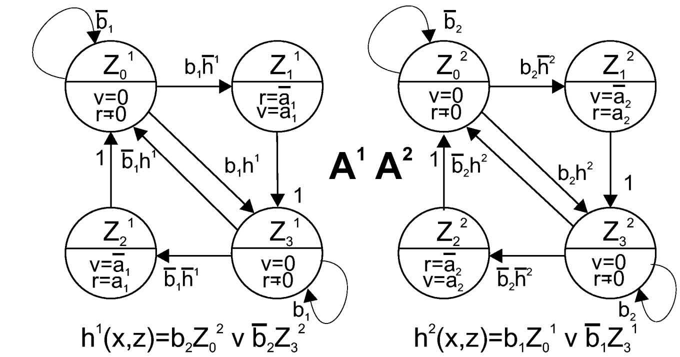

Figure 4: Example: net of automaton, system of FSM's

With a view on the coupling function of both graphs it can be ascertained that both graphs observe themselves only according their states. It concerns a regular structure. Interestingly enough one can find out and also prove that both graphs can be transferred by composition respectively decomposition into one another. The explicit description complexity rises nevertheless more slowly than the number of the implicitly noted global system states. But this implicit complexity can lead to semantic traps straightly, [2].

4 SEMANTIC TRAPS

For this purpose we regard again the general Mealy-structure from figure 1 and are coupling two such structures both according to their states and their outputs. That leads to the coupled structure in figure 5.

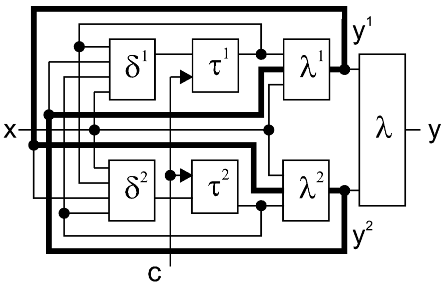

Figure 5: General Mealy-coupling

In the figure a structure is emphasized that was caused by the direct feedback of the λ^1 und λ^2 blocks. The expert recognizes immediately an additional asynchronous sequential structure. Note: This can happen only with Mealy-machine couplings, because the Moore-output function depends only on states and differs from the Mealy-output function where additionally a direct influence of the inputs is possible. We speak now about irregular structures. With this additional structure however the number of states after composition increases in comparison to the opposite product discussed above. In addition consider: The original automata were and are synchronous. The behavior of the whole system is asynchronous now which is represented in the practical design flow by a serious trap. Even the design of relative small systems can lead to such situations and without formal feedback analysis it is difficult to discover them, [3] .

In the interaction of the components the desired behavior (constructional knowledge) of the system of FSM's

$$z_k := \bigvee_{j \in M_1^k} \left(\bigvee_{i=0}^{2^n-1} k_i(z) h_{ij}(x) \right) \tag{7}$$

$$j \in M_1^k \leftrightarrow Z_j(z_k) = 1 \tag{8}$$

$$y_k = \bigvee_{i=0}^{2^n-1} k_i(z) h_{ij}(x) \tag{9}$$

can not be reached surely, and/or unwanted behavior (restrictive knowledge)

$$h^*(x,z) = \bigvee_{i=0}^{2^n-1} k_i(z) h_i^*(x) \tag{10}$$

can not be excluded surely.

The system of FSM's shown in figure 4 represents a mixture of constructional and restrictive knowledge.

- Constructional knowledge
  The output v is set to 1 in state Z_1^1 [1] with the rising edge of $b1$ and a high signal on $a1$ at the same time.

$$z_0^1 := \overline{z}_1^1 \overline{z}_0^1 b_1 \vee \ldots \tag{11}$$

$$v = \overline{z}_1^1 z_0^1 a_1 \vee \ldots \tag{12}$$

- Restrictive knowledge
  A simultaneous generation of v and r impulses have to be prevented.

$$h^* = \overline{z}_1^1 z_0^1 \overline{z}_1^2 z_0^2 b_1 \overline{h}^1 b_2 \overline{h}^2 \vee \ldots \tag{13}$$

The restrictive knowledge can be converted into constructional knowledge. In addition the transition terms of the components of the system of FSM's have to ensure the adherence to the restrictions. For example it is easy to examine that both automata can not be simultaneous in the states Z_1^1 and Z_1^2 at the same time.

Further semantic errors can not to be excluded. Interesting characteristics are for example
the completeness

$$\forall_i \left(\bigvee_{j=0}^{2^n-1} h_{ij}(x) = 1 \right), \tag{14}$$

the absence of conflicts

$$\forall_i \left(\bigvee_{j,l=0; j \neq l}^{2^n-1} h_{ij}(x) h_{il}(x) = 0 \right) \tag{15}$$

and the absence of deadlocks

$$\forall_{i,j,k,l,m,n; m \neq n \neq l} (Z_i^m Z_j^n \ldots Z_k^l \rightarrow h_{ii}^m h_{jj}^n \ldots h_{kk}^l = 1). \tag{16}$$

In the following section the absence of deadlocks is discussed more in detail. The state diagram fragments specify a parallel system, see figure 6.

[1] each state is coded by a certain number of variables, here $Z_1^1 : z_1^1 z_0^1$

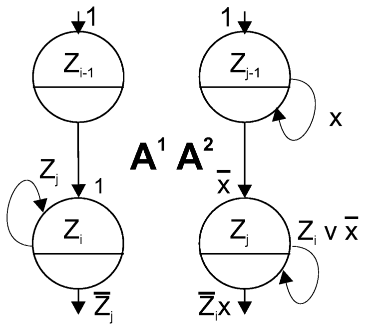

Figure 6: Example: Deadlock in a system of FSM's

In particular it is also called parallel because state transitions can be perfomed in parallel. The system can be simulated by a flow of marks. One variant can be the following for example: Based on the states Z_{i-1} and Z_{j-1} with $x = 0$ a parallel state transition in the system of FSM's occurs. Both marks are now in the states Z_i respectively Z_j. The FSM A^1 stops until FSM A^2 leaves its state and vice versa. That way both automata wait on one another. Both FSM's are blocked, because the conditions for leaving their states are never fulfilled. There exist other variants which do not lead into that blocked state. Generalized a blocking can occur. To obtain general statements about the system behavior of a parallel system the system function has to be explicitly noted. For this purpose a composition of the system of FSM's has to be accomplished.

$$\begin{aligned} Z_iZ_j \;\; := \;\; & Z_iZ_j(Z_iZ_j \vee Z_j\overline{x}) \vee Z_{i-1}Z_j(Z_i \vee \overline{x}) \\ & \vee Z_{i-1}Z_{j-1}\overline{x} \vee Z_iZ_{j-1}(Z_j\overline{x}) \end{aligned} \tag{17}$$

After the substitution and minimizing there follows:

$$Z_iZ_j := Z_iZ_j \vee Z_{i-1}Z_j\overline{x} \vee Z_{i-1}Z_{j-1}\overline{x}. \tag{18}$$

The result can be interpreted as a state diagram again. We receive a fragment of a global automaton, s. figure 7.

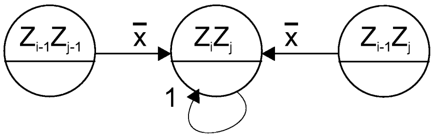

Figure 7: Example: graph fragment after composition

The transition term of the self-loop of state Z_iZ_j for the composition of the two automata fragments is 1. That means, there is no condition from this state that leads out, the system is in a deadlock. We had noted that a composition for quantitative reasons cannot be accomplished in every case. As a result of the investigation about deadlocks for a system of FSM's the following necessary [1] and sufficient condition for a deadlock can be indicated.

$$\begin{gathered} \forall i, j, k, m, n, l \in N; m \neq n \neq l \\ Z_i^m Z_j^n \dots Z_k^l \rightarrow h_{ii}^m h_{jj}^n \dots h_{kk}^l = 1 \end{gathered} \tag{19}$$

For the state Z_iZ_j the system of FSM's, see figure 6 the formula 19 becomes true.

$$\begin{aligned} Z_iZ_j \rightarrow Z_j(Z_i \vee \overline{x}) &= 1 \\ \overline{Z_iZ_j} \vee Z_iZ_j \vee Z_j\overline{x} &= 1 \\ 1 &= 1 \end{aligned} \tag{20}$$

The formula 19 applies to any number of components of a system of FSM's. A composition into a global automaton is not necessary. A deadlock analysis based on systems of FSM's and the formula 19 can be accomplished.

5 CONCLUSION

The main subject of interest was a theoretical analysis of semantic traps caused by the design of digital parallel systems. As formal description language finite state machines were used. Based on of the formula 19 an analysis of systems of FSM's for a deadlock analysis is possible. The coupling of Mealy-machine structures were of special interest. As a result of the investigations here is to ascertain, that a direct output coupling as shown in figure 5 is to avoid. With the help of the Designtool MLDesigner [4] different simulations for validation of the achieved knowledge were accomplished for a multiplicity of applications.

References

[1] Zerbe, V.; Spezifikation und HLL-Implementierung von Multitaskbetriebssytemen. TH Ilmenau, Dissertation, 1990

[2] Krapp, M.: Digitale Automaten. Verlag Technik Berlin, 1988

[3] Zerbe, V.: Systematischer Entwurf paralleler, digitaler Systeme. Workshop "Boolsche Probleme", Freiberg (Germany) October 07 1994

[4] MLDesigner: http://www.mldesigner.com